Exercise 12

Let $\mathbf{v} = (2, 3)$. Suppose $\mathbf{w} \in \mathbb{R}^2$ is perpendicular to $\mathbf{v}$, and that $\|\mathbf{w}\| = 5$. This determines $\mathbf{w}$ up to sign. Find one such $\mathbf{w}$.

Solution

Since $\mathbf{v} = (2, 3)$ and $\mathbf{w} = (w_x, w_y)$ are perpendicular, the dot product of these two vectors must be zero.

$$\begin{aligned} \mathbf{v} \cdot \mathbf{w} &= 0 \\ (2, 3) \cdot (w_x, w_y) &= 0 \\ 2w_x + 3w_y &= 0 \end{aligned} \tag{1}$$

The magnitude of $\mathbf{w}$ is known:

$$\|\mathbf{w}\| = \sqrt{w_x^2 + w_y^2} = 5. \tag{2}$$

Solve equations (1) and (2) for w_x and w_y.

$$w_x = \pm\frac{15}{\sqrt{13}} \quad \text{and} \quad w_y = \mp\frac{10}{\sqrt{13}}$$

Therefore, one such $\mathbf{w}$ is

$$\mathbf{w} = \left(\frac{15}{\sqrt{13}}, -\frac{10}{\sqrt{13}}\right).$$